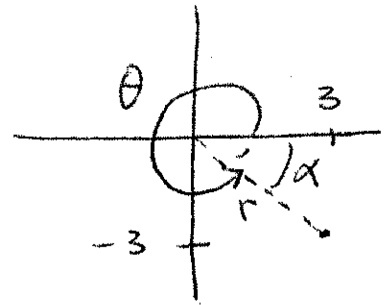
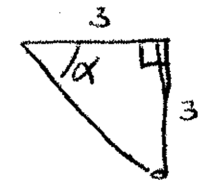
Q1

1(a) $(3, -3)$





$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

$x = 3 = r\cos\theta$

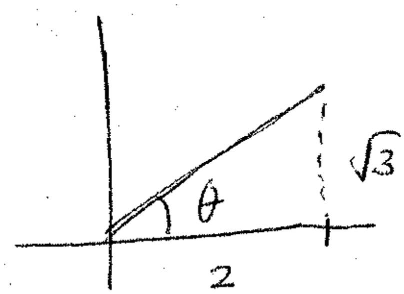
$y = -3 = r\sin\theta$

$x^2+y^2 = r^2 = 3^2+3^2 = 18 \Rightarrow r = \sqrt{18}$

$\tan\theta = \frac{y}{x} = -1 \Rightarrow \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$(\sqrt{18}, 7\pi/4)$ ~ polar coordinates.

1(b) $(2, \sqrt{3})$



$x = 2 = r\cos\theta$

$y = \sqrt{3} = r\sin\theta$

$x^2+y^2 = 4+3 = 7 = r^2 \Rightarrow r = \sqrt{7}$

$\tan\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 0.7137$ radians

$(\sqrt{7}, 0.7137)$

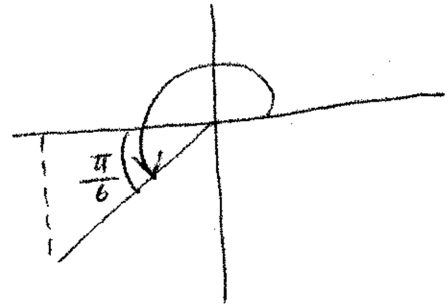
2(a) $(\sqrt{2}, \pi/4)$ ~ polar coordinates

$x = \sqrt{2}\cos\pi/4 = 1$

$y = \sqrt{2}\sin\pi/4 = 1$

$(1, 1)$ ~ rectangular coord

2(b) $(2, 7\pi/6) \sim$ polar

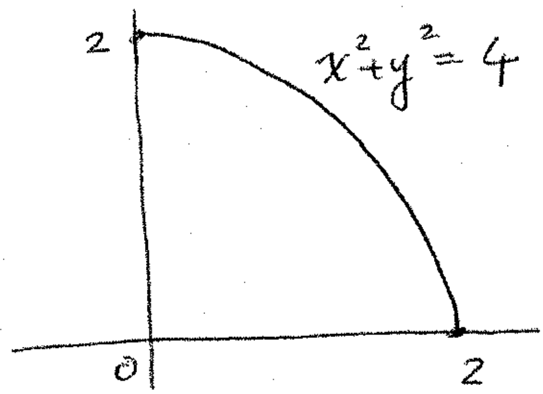


$x = 2\cos\frac{7\pi}{6} = 2\cdot\frac{-\sqrt{3}}{2} = -\sqrt{3}$

$y = 2\sin\frac{7\pi}{6} = 2\cdot\frac{-1}{2} = -1$

$(-\sqrt{3}, -1) \sim$ rectangular

Q3



$Q: \quad 0 \le r \le 2$
$\quad\;\; 0 \le \theta \le \pi/2$

in polar coordinates

$f(x,y) = 2x^2+y^2 = x^2 + x^2+y^2$

$f(r,\theta) = r^2\cos^2\theta + r^2 \leftarrow$ density in polar coordinates

$$\Delta M = f(r,\theta)\cdot \underbrace{\Delta A}_{r\Delta\theta\Delta r} = (r^2\cos^2\theta + r^2)\cdot r\cdot\Delta\theta\Delta r$$

$$= (r^3\cos^2\theta + r^3)\Delta\theta\Delta r$$

$$M = \iint_Q f(r,\theta)\, dA = \int_0^2\int_0^{\pi/2}(r^2\cos^2\theta + r^2)\,r\,d\theta\,dr$$

2

$$M = \int_0^2 \int_0^{\pi/2} (r^3\cos^2\theta + r^3)\, d\theta\, dr$$

$$= \int_0^2 \int_0^{\pi/2} \left[r^3\left(\frac{1+\cos 2\theta}{2}\right) + r^3\right] d\theta\, dr$$

$$= \int_0^2 \left[r^3\left(\frac{\theta + \frac{1}{2}\sin 2\theta}{2}\right) + r^3\theta\right]_0^{\pi/2} dr$$

$$= \int_0^2 \left[r^3\underbrace{\left(\frac{\frac{\pi}{2}+0}{2}\right)}_{\frac{\pi}{4}} + r^3 \cdot \frac{\pi}{2} - 0\right] dr$$

$$= \int_0^2 \frac{3\pi}{4} r^3\, dr = \frac{3\pi r^4}{16}\Bigg|_0^2 = \boxed{3\pi}\ \text{kg}$$

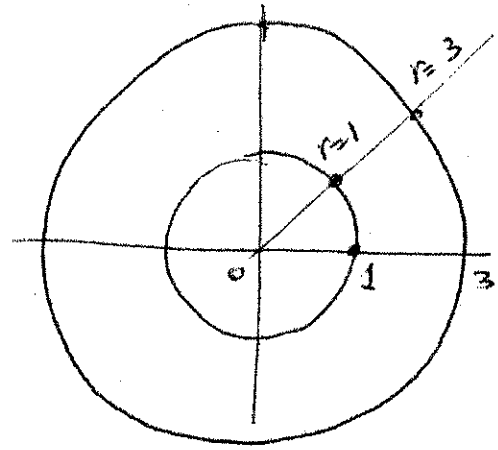
You can also do:

$$\int_0^{3\pi/2} \int_0^2 f(r,\theta) \cdot r\,dr\,d\theta$$

Q4

$R: \; 1 \le r \le 3$

$\quad 0 \le \theta \le 2\pi$



$$\Delta P = \underbrace{\rho(r,\theta)}_{\text{density}} \cdot \underbrace{\Delta A}_{\text{area}}$$

$$= \frac{8000 \sin^2(\theta/2)}{9+r^2} \cdot \Delta A$$

$$P = \iint_R \rho(r,\theta)\, dA = \int_{r=1}^{3} \int_{r=0}^{2\pi} \frac{8000 \sin^2(\theta/2)}{9+r^2} \cdot r\, d\theta\, dr$$

$$= \int_1^3 \int_0^{2\pi} \frac{8000r}{9+r^2} \underbrace{\left(\frac{1-\cos\theta}{2}\right)}_{\sin^2(\theta/2)} d\theta\, dr$$

$$\boxed{\begin{aligned} u &= 9+r^2 \\ du &= 2r\,dr \end{aligned}}$$

$$= \int_1^3 \left[\frac{8000r}{9+r^2}\left(\frac{\theta - \sin\theta}{2}\right)\right]_0^{2\pi} dr$$

$$= \int_1^3 \frac{8000r \cdot \pi}{9+r^2}\, dr = \int_{10}^{18} \frac{\cancel{8000}^{\,4000}\pi}{u} \cdot \frac{1}{\cancel{2}}\, du$$

4

$$= 4000\pi\left[\ln 18 - \ln 10\right] = 4000\pi \ln(1.8)$$